Parametric Time-domain models based on frequency-domain data

(Module 7)

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Time-domain modelling approaches

Two approaches can be distinguished for time-domain modelling:

- Full time-domain hydrodynamic codes,
- Time-domain models based on frequency-domain data.

Here, we will focus on the second approach.

Frequency-domain Eq. of Motion

In the hydrodynamic literature it is common to find the following model:

$$[\mathbf{M}_{RB} + \mathbf{A}(\omega)]\ddot{\boldsymbol{\xi}}(t) + \mathbf{B}(\omega)\dot{\boldsymbol{\xi}}(t) + \mathbf{G}\boldsymbol{\xi}(t) = \boldsymbol{\tau}_{exc}(t)$$

- This is **not a true time-domain** model (Cummins, 1962)
- This is valid to describe the steady-state response to sinusoidal excitations—i.e., **Frequency Response**:

$$\tilde{\boldsymbol{\xi}} = \underbrace{\left(-\omega^2[\mathbf{M}_{RB} + \mathbf{A}(\omega)] + j\omega\,\mathbf{B}(\omega) + \mathbf{G}\right)^{-1}}_{\textit{Force to Motion RAO}}\tilde{\boldsymbol{\tau}}_{exc}$$

~ denotes complex variable

Cummins's equation

Cummins (1962), took a different modelling approach and consider the radiation problem *ab initio* in the time domain :

$$[\mathbf{M}+\mathbf{A}]\,\ddot{\xi}+\int_0^t \mathbf{K}(t-\tau)\,\dot{\xi}(\tau)\,d\tau+\mathbf{G}\,\xi=\boldsymbol{\tau}_w$$

- The added mass matrix is constant—frequency and speed independent.
- The convolution term accounts for fluid-memory effects.
- The kernel of the convolution is a matrix of retardation functions or impulse responses.
- This is a true linear time-domain model.

Cummins's equation with forward speed

$$[\mathbf{M}+\mathbf{A}]\,\ddot{\xi}+\mathbf{B}(U)\,\dot{\xi}+\int_0^t \mathbf{K}(t-\tau,U)\,\dot{\xi}(\tau)\,d\tau+[\mathbf{G}+\mathbf{G}'(U)]\,\xi=\boldsymbol{\tau}_w.$$

- The convolution terms depend on the forward speed.
- With forward speed appears a constant damping term.
- The restoring forces are affected by hydrodynamic pressure—Lift, changes in trim. (Usually ignored for Fn<0.3)

Ogilvie's relations

If Cummins's Equation is valid for any input, it must then be valid for sinusoids in particular (Ogilvie, 1964).

Ogilvie (1964) transformed the Cummins' Equation to the frequency domain, and found that

$$\mathbf{A}(\omega) = \mathbf{A} - \frac{1}{\omega}\int_0^\infty \mathbf{K}(t)\sin(\omega t)\,dt$$

$$\mathbf{B}(\omega) = \mathbf{B}(U) + \int_0^\infty \mathbf{K}(t)\cos(\omega t)\,dt$$

From the Riemann-Lesbesgue Lemma:

$$\mathbf{A} = \lim_{\omega\to\infty} \mathbf{A}(\omega) := \mathbf{A}(\infty)$$

$$\mathbf{B}(U) = \lim_{\omega\to\infty} \mathbf{B}(\omega) := \mathbf{B}(\infty)$$

Non-parametric Representations

- Time-domain

$$\mathbf{K}(t) = \frac{2}{\pi}\int_0^{\infty} [\mathbf{B}(\omega) - \mathbf{B}(\infty)]\cos(\omega t)\, d\omega$$

- Frequency-domain

$$\mathbf{K}(j\omega) = [\mathbf{B}(\omega) - \mathbf{B}(\infty)] + j\omega[\mathbf{A}(\omega) - \mathbf{A}(\infty)]$$

Parametric Representations

Because the convolution is a dynamic linear operation, it can be represented by a linear ordinary differential equation—state-space model:

$$\boldsymbol{\mu}_r = \int_0^\infty \mathbf{K}(t-\tau)\,\dot{\boldsymbol{\xi}}(\tau)\,d\tau \quad \Leftrightarrow \quad \begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}_c\mathbf{x} + \mathbf{B}_c\dot{\boldsymbol{\xi}} \\ \boldsymbol{\mu}_r &= \mathbf{C}_c\mathbf{x} \end{aligned} \qquad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

Parametric Representations

From the state-space representation, it follow

- Impulse Response

$$\mathbf{K}(t) = \mathbf{C}_c \exp(\mathbf{A}_c t)\mathbf{B}_c$$

- Frequency-domain model (Transfer Function matrix)

$$\mathbf{K}(s) = \mathbf{C}_c(s\mathbf{I} - \mathbf{A}_c)\mathbf{B}_c = \begin{bmatrix} \frac{P_{11}(s)}{Q_{11}(s)} & \cdots & \frac{P_{16}(s)}{Q_{16}(s)} \\ \vdots & \ddots & \vdots \\ \frac{P_{61}(s)}{Q_{61}(s)} & \cdots & \frac{P_{66}(s)}{Q_{66}(s)} \end{bmatrix}$$

$$\frac{P_{ij}(s)}{Q_{ij}(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_0}{s^n + q_{n-1} s^{n-1} + \cdots + q_0}$$

Rational TF

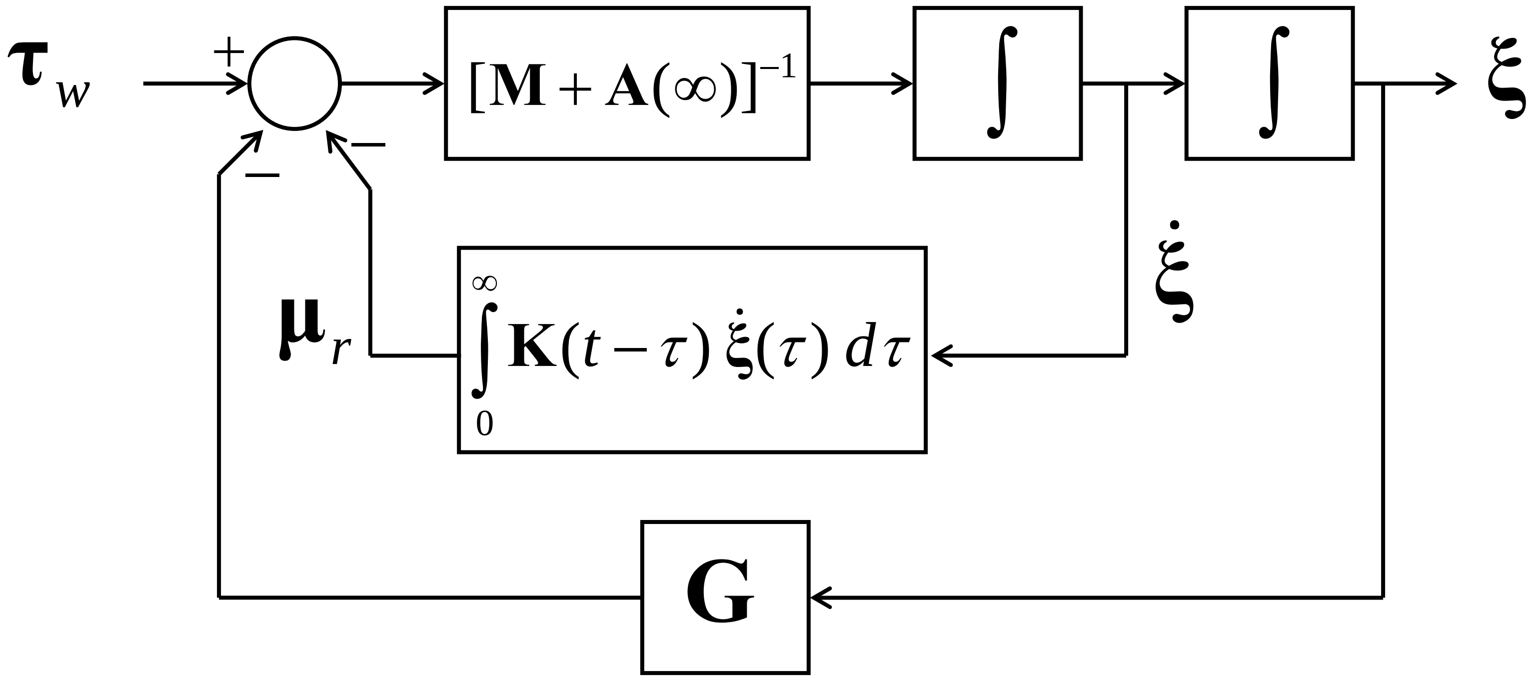
Convolution replacement

- The convolution (non-parametric model) in the Cummins equation can be time and memory consuming for simulation.

- For analysis and design of a control system, the convolutions are not very well suited.

- The parametric state-space representation of appropriate order (n-order) eliminates the above problems.

Convolution replacement



$$\boldsymbol{\mu}_r = \int_0^\infty \mathbf{K}(t-\tau)\,\dot{\boldsymbol{\xi}}(\tau)\,d\tau \quad \Leftrightarrow \quad \begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}_c\mathbf{x} + \mathbf{B}_c\dot{\boldsymbol{\xi}} \\ \boldsymbol{\mu}_r &= \mathbf{C}_c\mathbf{x} \end{aligned}$$

Properties of the convolution terms

Property	Implication on parametric models
$\lim_{\omega \to 0} \mathbf{K}(j\omega) = -\mathbf{B}(\infty)$	$\mathbf{K}$(s) is zero at s=0 for U=0.
$\lim_{\omega \to \infty} \mathbf{K}(j\omega) = \mathbf{0}$	TFs strictly proper
$\mathbf{K}(t = 0^+) = \int_0^\infty [\mathbf{B}(\omega) - \mathbf{B}(\infty)] d\omega \neq \mathbf{0}$	TFs relative degree 1
$\lim_{t \to \infty} \mathbf{K}(t) = \mathbf{0}$	TF BIBO stable
$\mathrm{Re}\{K_{ii}(j\omega)\} \geq 0$	$\mathbf{K}$(s) is passive => diagonal terms are positive real; off diagonal terms stable.

Note: **Bold** symbols denote matrices.

Low-frequency limit (U=0)

$$\mathbf{K}(j\omega) = [\mathbf{B}(\omega) - \mathbf{B}(\infty)] + j\omega[\mathbf{A}(\omega) - \mathbf{A}(\infty)]$$

In the limit at low freq, **B**(ω) is zero, since there cannot be waves (Faltinsen, 1990); thus the real part is zero for U=0 and -**B**(∞) for U>0.

The imaginary part tends to zero as the following difference is finite:

$$\mathbf{A}(0) - \mathbf{A}(\infty) = \lim_{\omega \to 0} \frac{-1}{\omega} \int_0^\infty \mathbf{K}(t) \sin(\omega t)\, dt = -\int_0^\infty \mathbf{K}(t) \lim_{\omega \to 0} \frac{\sin(\omega t)}{\omega}\, dt = -\int_0^\infty \mathbf{K}(t)\, dt$$

Note that regularity conditions for the exchange of limit and integration are satisfied.

High-frequency limit (U=0)

$$\mathbf{K}(j\omega) = [\mathbf{B}(\omega) - \mathbf{B}(\infty)] + j\omega[\mathbf{A}(\omega) - \mathbf{A}(\infty)]$$

In the limit at high frequency, the real part is zero.

The imaginary part also tends to zero by Ogilvie's relation and Riemann-Lebesgue Lemma:

$$\lim_{\omega \to \infty} \omega[\mathbf{A}(0) - \mathbf{A}(\infty)] = \lim_{\omega \to \infty} \int_0^\infty -\mathbf{K}(t)\sin(\omega t)\, dt = \mathbf{0}$$

Initial and final time (U=0)

- Initial time:

$$\lim_{t\to 0^+}\mathbf{K}(t)=\lim_{t\to 0^+}\frac{2}{\pi}\int_0^\infty[\mathbf{B}(\omega)-\mathbf{B}(\infty)]\cos(\omega\, t)\, d\omega=\frac{2}{\pi}\int_0^\infty[\mathbf{B}(\omega)-\mathbf{B}(\infty)]\, d\omega\neq\mathbf{0}$$

  - Regularity conditions for the exchange of limit and integration are satisfied.
  - The last relation follows from energy considerations (Faltinsen, 1990):

- Final time:

$$\lim_{t\to\infty}\mathbf{K}(t)=\lim_{t\to\infty}\frac{2}{\pi}\int_0^\infty[\mathbf{B}(\omega)-\mathbf{B}(\infty)]\cos(\omega\, t)\, d\omega=\mathbf{0}$$

Which follows by Ogilvie's relation and Riemann-Lebesgue Lemma.

Passivity

For U=0 and no current, the damping matrix is symmetric and positive semi-definite:

$$B(\omega) = B^T(\omega) \geq 0$$

From this follows the positive realness of the convolution terms and thus the passivity; that is these terms cannot generate energy.

From energy considerations, it also follows that the diagonal terms of **K**(s) are passive.

Parametric model identification

The convolution replacement can be posed in different ways, which in "theory" should provide the same answer:

Time-domain identification

$$\mathbf{B}(\omega) \Rightarrow \mathbf{K}(t) \Rightarrow \begin{bmatrix} \hat{\mathbf{A}}_c & \hat{\mathbf{B}}_c \\ \hat{\mathbf{C}}_c & \hat{\mathbf{D}}_c \end{bmatrix}$$

Frequency-domain identification

Model conversion

$$\mathbf{A}(\omega), \mathbf{B}(\omega) \Rightarrow \mathbf{K}(j\omega) \Rightarrow \hat{\mathbf{K}}(s) \Rightarrow \begin{bmatrix} \hat{\mathbf{A}}_c & \hat{\mathbf{B}}_c \\ \hat{\mathbf{C}}_c & \hat{\mathbf{D}}_c \end{bmatrix}$$

In practice one method can be more favourable than the other.

Parametric model identification

Different proposals have appeared in the literature:

Time-domain identification:

- LS-fitting of the impulse response (Yu & Falnes, 1998)
- Realization theory (Kristiansen & Egeland, 2003)

Frequency-domain identification:

- LS-fitting of the frequency response $K(j\omega)$ (Jeffreys, 1984),(Damaren 2000).
- LS-fitting of added mass and damping (Soding 1982), (Xia et. al 1998), (Sutulo & Guedes-Soares 2006).

Time-domain identification

From $\mathbf{K}(t)$ to state-space models.

Numerical computations of K(t)

A key issue for time-domain identification is to start with a good impulse response computed from the damping.

- Numerical codes can only provide accurate computations of added mass and damping up to a certain frequency, say Ω.
- This introduces an error in the computation of the retardation functions:

$$\mathbf{K}(t,U) = \frac{2}{\pi}\int_0^{\Omega}[\mathbf{B}(\omega) - \mathbf{B}(U)]\cos(\omega t)d\omega + \frac{2}{\pi}\int_{\Omega}^{\infty}[\mathbf{B}(\omega) - \mathbf{B}(U)]\cos(\omega t)d\omega$$

$$\mathbf{K}(t,U) \approx \frac{2}{\pi}\int_0^{\Omega}[\mathbf{B}(\omega) - \mathbf{B}(U)]\cos(\omega t)d\omega \qquad \mathbf{Error}(t,U) = \frac{2}{\pi}\int_{\Omega}^{\infty}[\mathbf{B}(\omega) - \mathbf{B}(U)]\cos(\omega t)d\omega$$

High-frequency values of A(ω) and B(ω)

In the limit at high frequency the following tendencies are observed for the 3D damping and added mass:

$$B_{ik}(\omega) \propto \frac{\alpha_{ik}}{\omega^2} \quad as\ \omega \to \infty \qquad\qquad A_{ik}(\omega) - A_{ik}(\infty) \propto \frac{\beta_{ik}}{\omega^2} \quad as\ \omega \to \infty$$

As commented by Damaren (2000), this seems at odds with what is generally stated in the hydrodynamic literature!

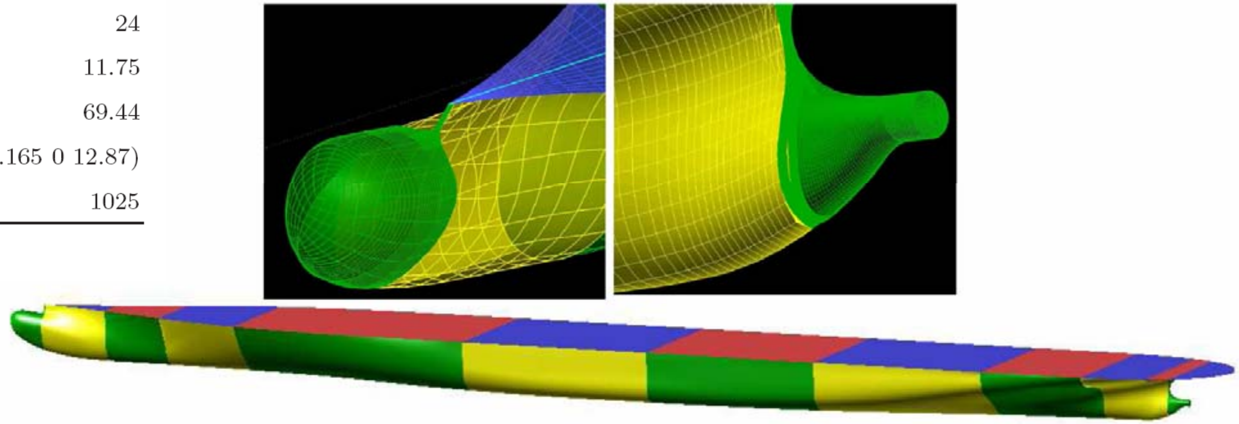
Note that there are no expansions involved to obtain these results, the only assumption is the linearity which results in a rational representation and the relative degree 1, which results from the integration of damping over the frequencies.

Example Containership (Taghipour et al., 2007a)

Vessel specifications

Quantity	Dimension	Value
Mass	kg	7.6656E7
Length overall	m	294.008
Beam	m	32.26
Height	m	24
Draft	m	11.75
Pitch gyration radius	m	69.44
C_G coordinates [a]	m	(-4.165 0 12.87)
Water Density	kg/m^3	1025

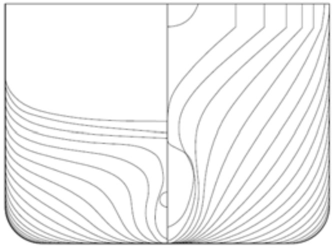
[a] measured from midships and keel



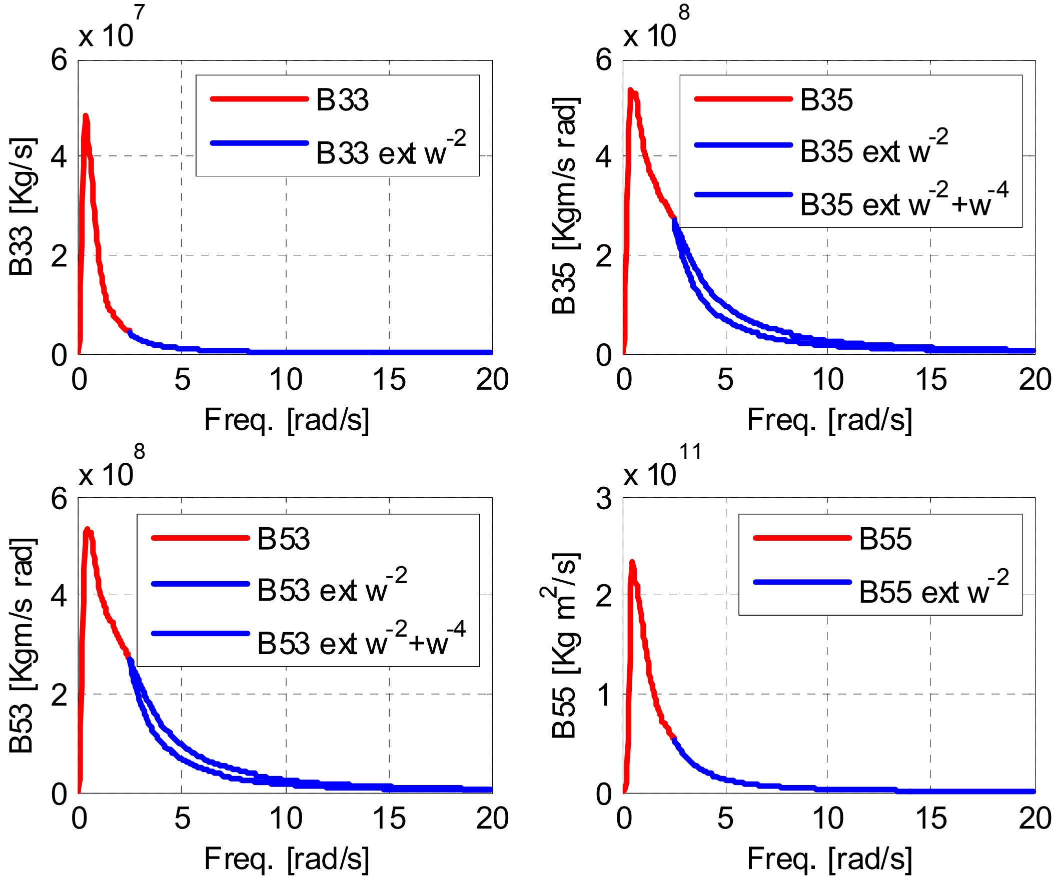
The panel sizing was done to be able to compute frequencies up to 2.5 rad/s.

Rule of thumb: characteristic panel length < 1/8 min wave length (Faltinsen, 1993).

Numerical computations



Extending the damping with tail prop to $1/\omega^2$

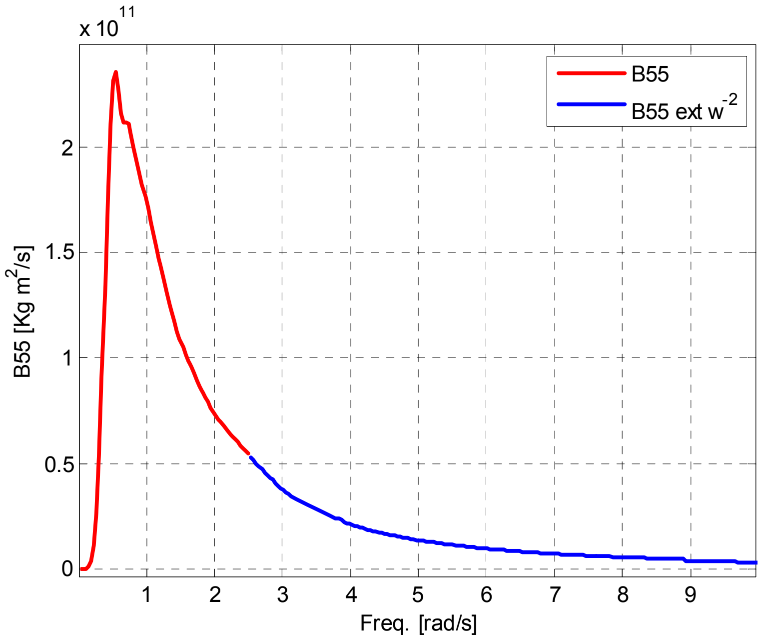


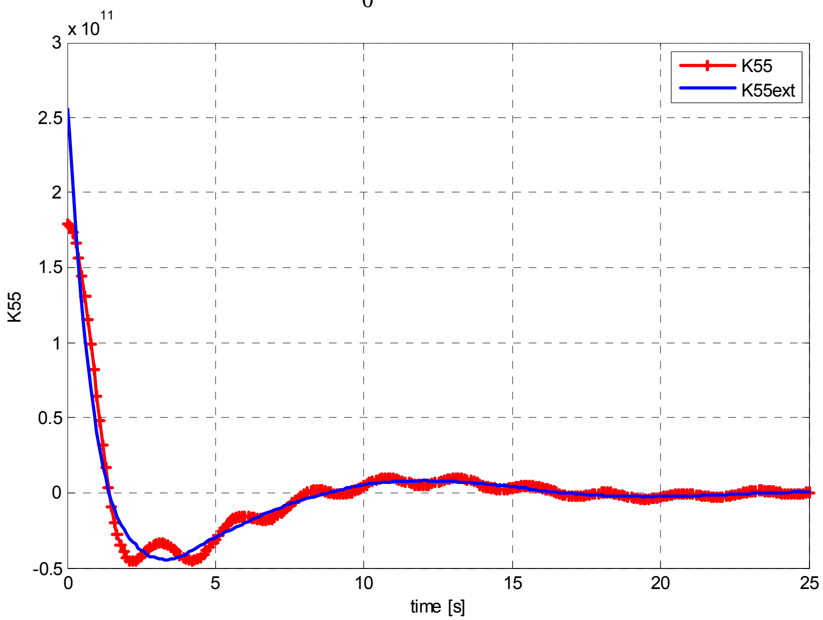
In this example, the tail α/ω^2 is not a very good for B35 and B53 (2.5rad/s is too low), whereas it is ok for B33 and B55.

Numerical computations

Sometimes it is necessary to extend the damping with asymptotic values:

$$\mathbf{K}(t) \approx \frac{2}{\pi}\int_0^{\Omega} \mathbf{B}_{ext}(\omega)\cos(\omega t)d\omega$$





When doing time-domain identification, it is important to start from a good approx of the retardation function!

Note the differences at $t=0^+$, and the errors at different time instants.

Impulse response curve fitting

Given the SISO SS realization of order n:

$$\dot{\mathbf{z}}(t) = \mathbf{A}'(\boldsymbol{\theta})\,\mathbf{z}(t) + \mathbf{B}'(\boldsymbol{\theta})\,u(t)$$
$$y(t) = \mathbf{C}'(\boldsymbol{\theta})\,\mathbf{z}(t),$$

The parameters can be obtained from

$$\boldsymbol{\theta}^{\star} = \arg\min_{\boldsymbol{\theta}} \sum_i w_i \left| h(t_i) - \hat{h}(t_i, \boldsymbol{\theta}) \right|^2$$

$$\hat{h}(t, \boldsymbol{\theta}) = \hat{\mathbf{C}}'(\boldsymbol{\theta}) \exp\{\hat{\mathbf{A}}'(\boldsymbol{\theta})\, t\}\, \hat{\mathbf{B}}'(\boldsymbol{\theta}),$$

The application of this method to marine structures was proposed by Yu and Falnes (1998).

Impulse response curve fitting

- It is hard to guess the order of the system by looking at the impulse response alone—one should start with lower order and increase it to improve the fit.

- The LS-problem is non-linear in the parameters. This problem can be solved with Gaussian-Newton methods.

- The Gaussian-Newton methods are known to work well if the parameters' initial guess are close to the optimal parameters.

- Since the initial value of the parameters is difficult to obtain from the impulse response, and these depends on the particular realization being chosen, the method is not very practical.

Realization theory

A key result of realization theory is the following factorization (Ho and Kalman, 1966):

$$\mathcal{H}_k = \begin{bmatrix} h_1 & h_2 & \dots & h_k \\ h_2 & h_3 & \dots & h_{k+1} \\ \vdots & \vdots & & \vdots \\ h_k & h_{k+1} & \dots & h_{2k-1} \end{bmatrix}$$

Hankel matrix of the impulse response values (constant along the anti-diagonals)

$$= \begin{bmatrix} \mathbf{C}' \\ \mathbf{C}'\mathbf{A}' \\ \mathbf{C}'\mathbf{A}'^2 \\ \vdots \\ \mathbf{C}'\mathbf{A}'^{k-1} \end{bmatrix} \begin{bmatrix} \mathbf{B}' & \mathbf{A}'\mathbf{B}' & \mathbf{A}'^2\mathbf{B}' & \dots & \mathbf{A}'^{k-1}\mathbf{B}' \end{bmatrix}$$

Extended controllability matrix

Extended observability matrix

Realization theory

Kung's Algorithm (Kung, 1978):

$$\mathcal{H}_k = \mathbf{U}\Sigma\mathbf{V}^*$$

Singular value decomposition

$$\Sigma = \begin{bmatrix} \sigma_1 & 0 & \dots & \\ 0 & \sigma_2 & \dots & \\ 0 & & \ddots & \\ & & 0 & \sigma_k \end{bmatrix} = \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix}$$

The number of significant singular values give the order of the system: $\Sigma_1 = n \times n$

$$\mathcal{H}_k = [\mathbf{U}_1 \mathbf{U}_2] \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} [\mathbf{V}_1^* \mathbf{V}_2^*] = \mathbf{U}_1 \Sigma_1 \mathbf{V}_1^*$$

Realization theory

Kung's Algorithm (Kung, 1978):

$$\mathbf{A}'_d = \Sigma_1^{-1/2} \begin{bmatrix} \mathbf{U}_{11} \\ \mathbf{U}_{12} \end{bmatrix}^T \begin{bmatrix} \mathbf{U}_{12} \\ \mathbf{U}_{13} \end{bmatrix} \Sigma_1^{1/2}$$

$$\mathbf{B}'_d = \Sigma_1^{-1/2} \mathbf{V}_{11}^*$$

$$\mathbf{C}'_d = \mathbf{U}_{11} \Sigma_1^{1/2}$$

$$\mathbf{D}'_d = h(0),$$

$$\mathbf{U}_1 = \begin{bmatrix} \mathbf{U}_{11} \\ \mathbf{U}_{12} \\ \mathbf{U}_{13} \end{bmatrix}, \quad \mathbf{V}_1 = \begin{bmatrix} \mathbf{V}_{11} \\ \mathbf{V}_{12} \\ \mathbf{V}_{13} \end{bmatrix}$$

$\mathbf{U}_{ii}$ and $\mathbf{V}_{ii}$ being $n \times n$

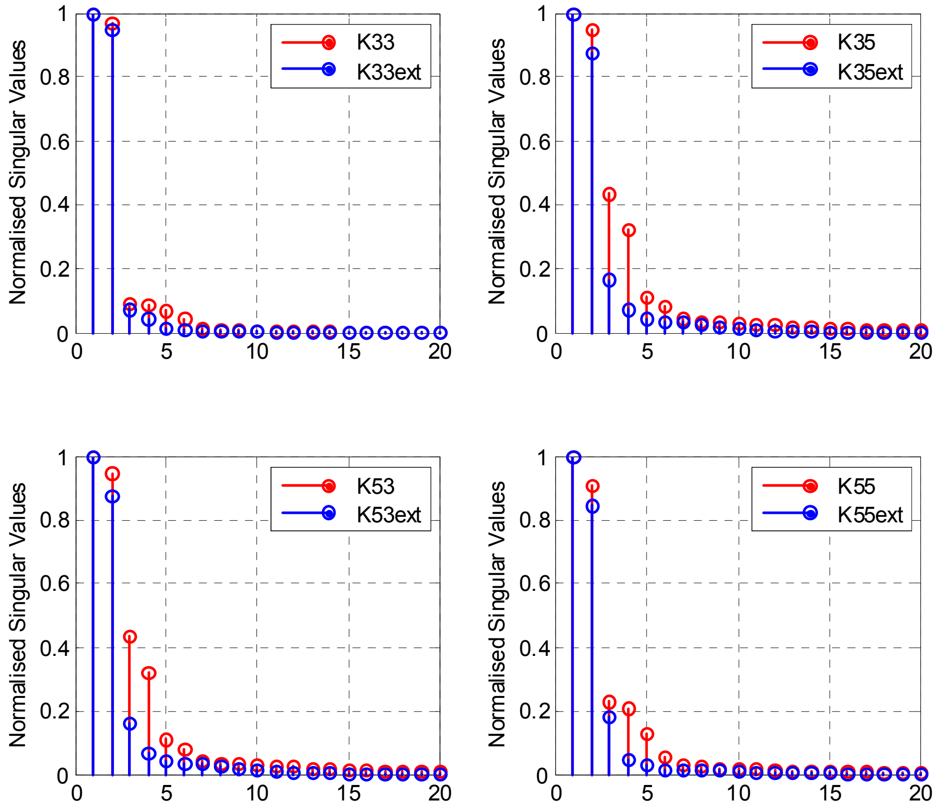
The application of this method to marine structures was proposed by Kristiansen and Egeland (2003).

Realization theory

- The problem is solved in discrete time
- Kung's algorithm obtains the model based on a SVD-factorization of the Hankel matrix of samples of the impulse response.
- If the impulse response is not accurate, it may in very large order systems
- The conversion from discrete to continuous often gives a matrix $\mathbf{D}_{c}$ in the state-space realization, which is inconsistent with the dynamics of the problem for the retardation function (relative degree 1).
- The MATLAB command imp2ss implements Kung's algorithm, and chooses the order by neglecting singular values less than 1% of the largest one.
- imp2ss requires using model order reduction afterwards (Kristiansen et al 2005).
- The resulting models may not be passive.

Example Container ship

Singular values of the Hankel matrix of the samples of the impulse response.



These suggest (based on the blue plots)

Order K33(s) = 3 or 4

Oder K35(s) = 5 or 6

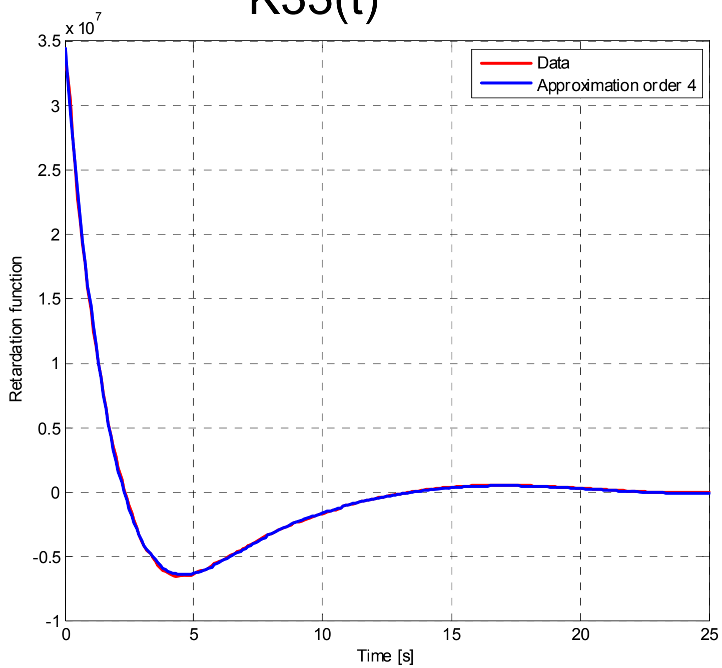
Order K55(s) = 3 or 4

Example Container ship

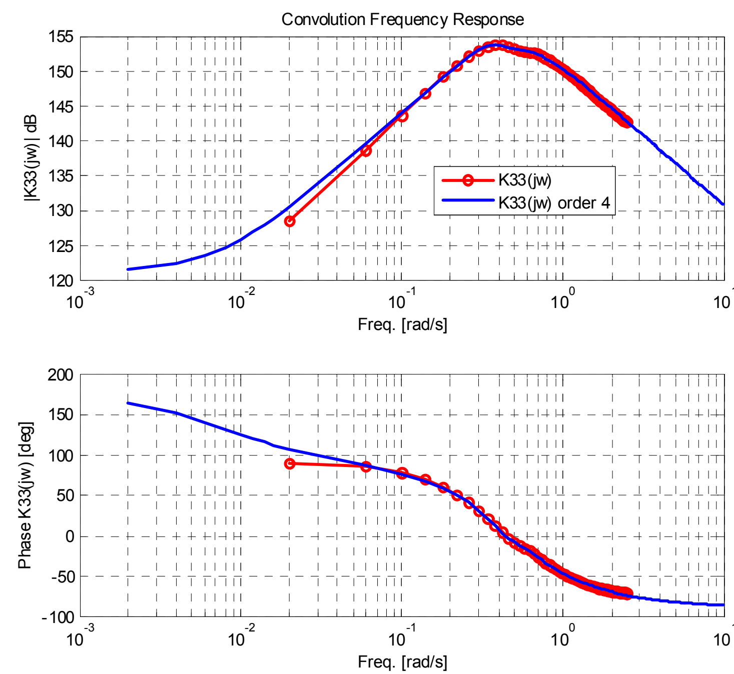
Impulse response fitting for K33(t) with a system of order 4.

Identification method: imp2ss + balmr (model order reduction).

K33(t)



K33(jw)



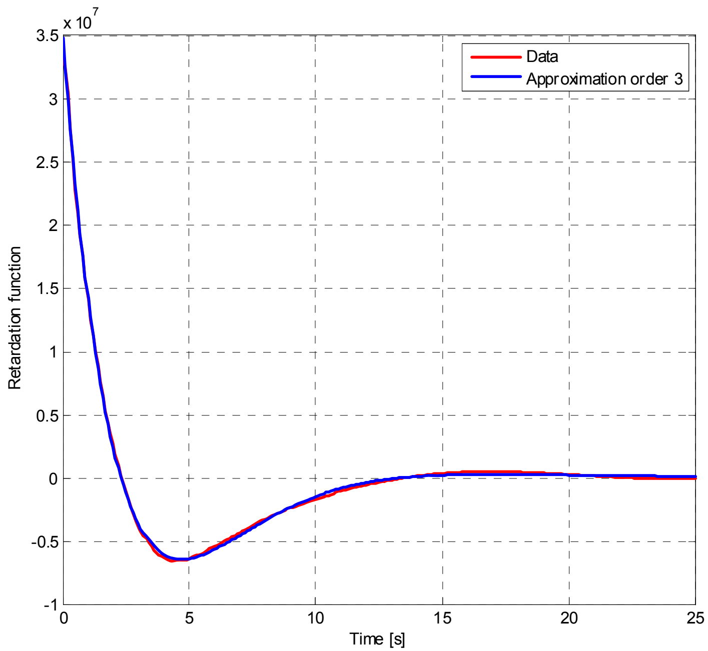
The identified model is not passive and does not have a zero at s=0.

Example Container ship

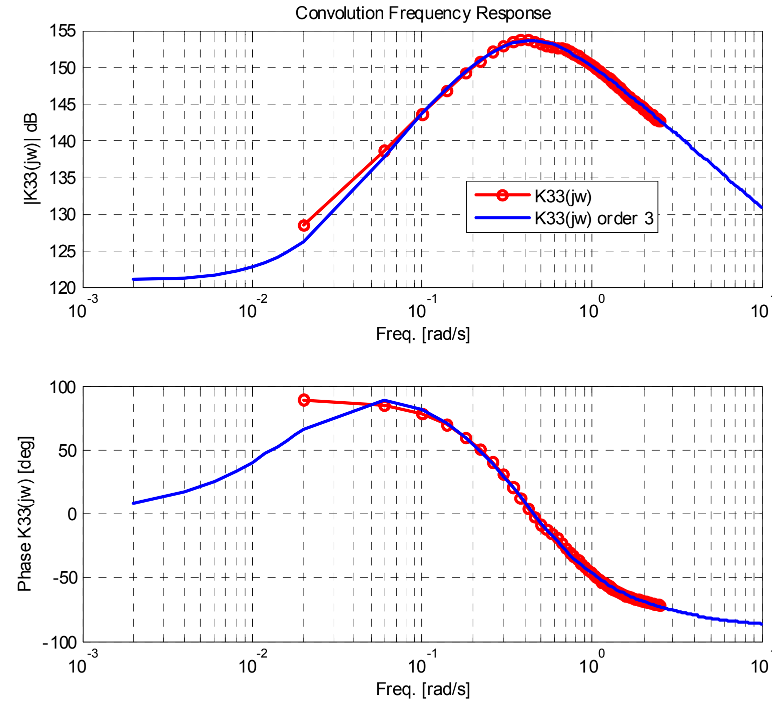
Impulse response fitting for K33(t) with a system of order 3.

Identification method: imp2ss + balmr (model order reduction).

K33(t)



K33(jw)

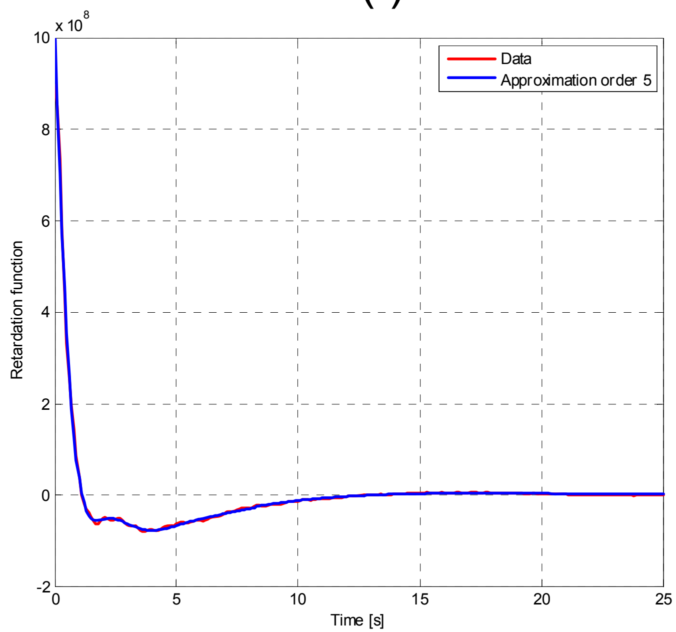


The identified model is passive, but still does not have a zero at s=0.

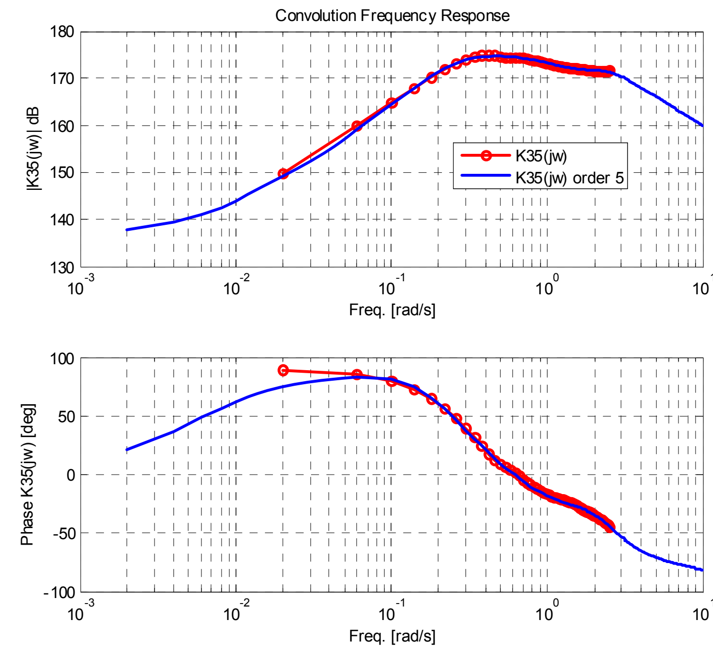
Example Container ship

Impulse response fitting for K35(t) with a system of order 5.

K35(t)



K35(jw)



The identified model is passive, but still does not have a zero at s=0.

Note that the off-diagonal terms do not necessarily have to passive for **K**(s) to be passive (Unneland & Perez, 2007).

Comments about Realization Theory

- Depending on the hydrodynamic data, it may be necessary to extend the damping at high freq. to have a good estimate the of the impulse response function before doing the idetification.
- Looking at the impulse response fitting alone is not a good criteria—most properties are evident from the freq. response.
- Imp2ss may require using model order reduction afterwards.
- The models almost never satisfy the low frequency asymptotic values (zero at s=0).
- High-order models may not be passive; this can solved trying different orders or using a model order reduction method that enforces passivity.

Frequency-domain Identification

From **K**($j\omega$) to **K**(s)

Frequency-domain identification

We can fit a SISO TF to each entry of $\mathbf{K}(j\omega)$:

$$\hat{K}_{ij}(s) = \frac{P_{ij}(s)}{Q_{ij}(s)}$$

Where,

- $P_{ij}(s)$ has a zero at s=0 for U=0 or constant for U>0
- Relative degree = 1; i.e., $\deg Q_{ij}(s) = \deg P_{ij}(s) + 1$
- Stable

Relative degree condition

From the finite initial time of the impulse response:

$$\lim_{t\to 0^+} K_{ij}(t) = \lim_{t\to 0^+} \frac{2}{\pi}\int_0^\infty [B_{ij}(\omega) - B_{ij}(\infty)]\cos(\omega\, t)\, d\omega = \frac{2}{\pi}\int_0^\infty [B_{ij}(\omega) - B_{ij}(\infty)]\, d\omega \neq 0$$

From the Initial-value Theorem of the Laplace Transform:

$$\lim_{t\to 0^+} K_{ij}(t) = \lim_{s\to\infty} sK_{ij}(s) = \lim_{s\to\infty} \frac{sP_{ij}(s)}{Q_{ij}(s)} = \frac{b_m s^{m+1}}{s^n}.$$

Hence for this to be finite n=m+1; relative degree = 1.

Minimum order transfer function

Because of the restriction of relative degree 1, the minimum order TF that can represent a convolution term is

$$K_{ij}^{\min}(s) = \begin{cases} \dfrac{b_1 s}{s^2 + a_1 s + a_0} & U = 0, \\[2ex] \dfrac{b_1 s + b_0}{s^2 + a_1 s + a_0} & U > 0, \end{cases}$$

Therefore, we can start with a system of order n=2, and then increase the order until we improve the fitting at an appropriate level.

Regression in the frequency domain

In this method, a rational transfer function

$$\hat{H}(s,\boldsymbol{\theta}) = \frac{P(s,\boldsymbol{\theta})}{Q(s,\boldsymbol{\theta})} = \frac{p_m s^m + p_{m-1} s^{m-1} + \ldots + p_0}{s^n + q_{n-1} s^{n-1} + \ldots + q_0}$$

$$\boldsymbol{\theta} = [p_m, \ldots, p_0, q_{n-1}, \ldots, q_0]^T$$

is fitted to the frequency response data:

$$\boldsymbol{\theta}^{\star} = \arg\min_{\boldsymbol{\theta}} \sum_i w_i \left| H(j\omega_i) - \frac{P(j\omega_i,\boldsymbol{\theta})}{Q(j\omega_i,\boldsymbol{\theta})} \right|^2$$

The application of this method to marine structures was proposed by Jeffreys (1984) and Damaren (2000).

Quasi-linear regression

Levi (1959) proposed the following linearization:

$$\boldsymbol{\theta}^\star = \arg\min_{\boldsymbol{\theta}} \sum_i |Q(j\omega_i, \boldsymbol{\theta})H(j\omega_i) - P(j\omega_i, \boldsymbol{\theta})|^2$$

This can be obtained If we chose the weights in the nonlinear problem as

$$w_i = |Q(j\omega_i, \boldsymbol{\theta})|^2$$

$$\boldsymbol{\theta}^\star = \arg\min_{\boldsymbol{\theta}} \sum_i w_i \left| H(j\omega_i) - \frac{P(j\omega_i, \boldsymbol{\theta})}{Q(j\omega_i, \boldsymbol{\theta})} \right|^2$$

which is affine in the parameters and reduces to a linear LS problem.

Iterative Quasi-linear regression

The quasi-linear regressor tend to have a poor fit a low freq. This can be avoided by solving the linear LS problem iteratively, starting with the quasi-linear regressor and using the parameters obtained to compute a weighing:

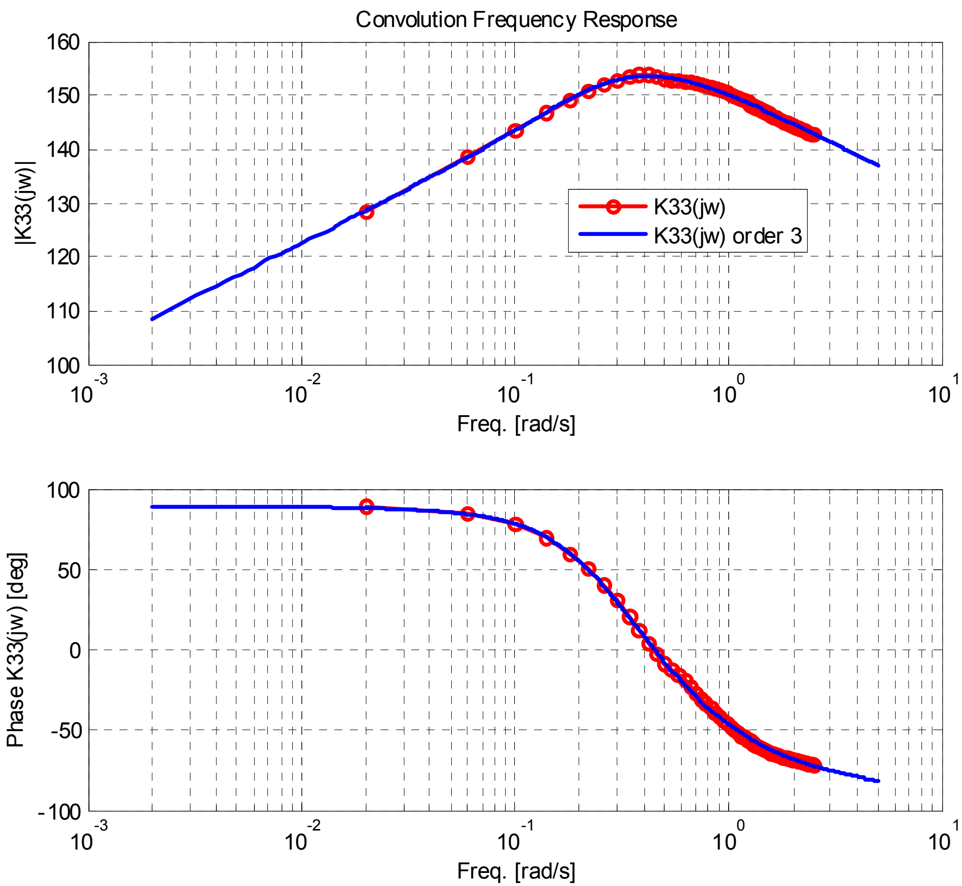
$$\boldsymbol{\theta}_k = \arg\min_{\boldsymbol{\theta}} \sum_i s_{i,k} \left| Q(j\omega_i, \boldsymbol{\theta}) H(j\omega_i) - P(j\omega_i, \boldsymbol{\theta}) \right|^2$$

$$s_{i,k} = \frac{1}{|Q(j\omega_i, \boldsymbol{\theta}_{k-1})|^2} \qquad k = 2,3,\ldots$$

After a few iterations, $Q(j\omega_i, \boldsymbol{\theta}_k) \approx Q(j\omega_i, \boldsymbol{\theta}_{k-1})$ and the nonlinear problem is recovered.

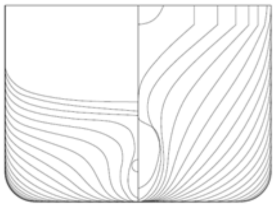
Containership example

Frequency-domain identification of K33(jw) order 3
Identification method: iterative quasi-linear regression (invfreqs.m)

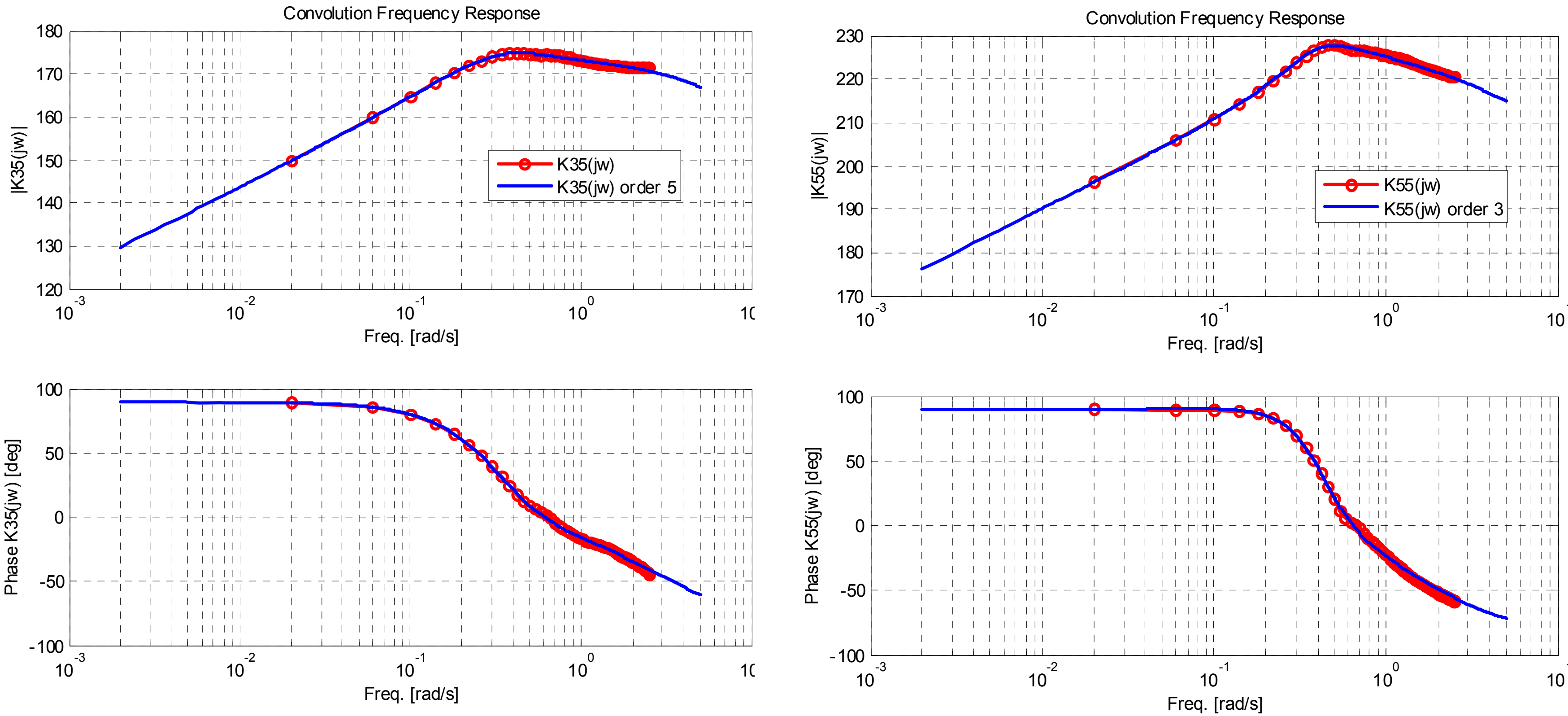


The model is passive and satisfy the asymptotic values.

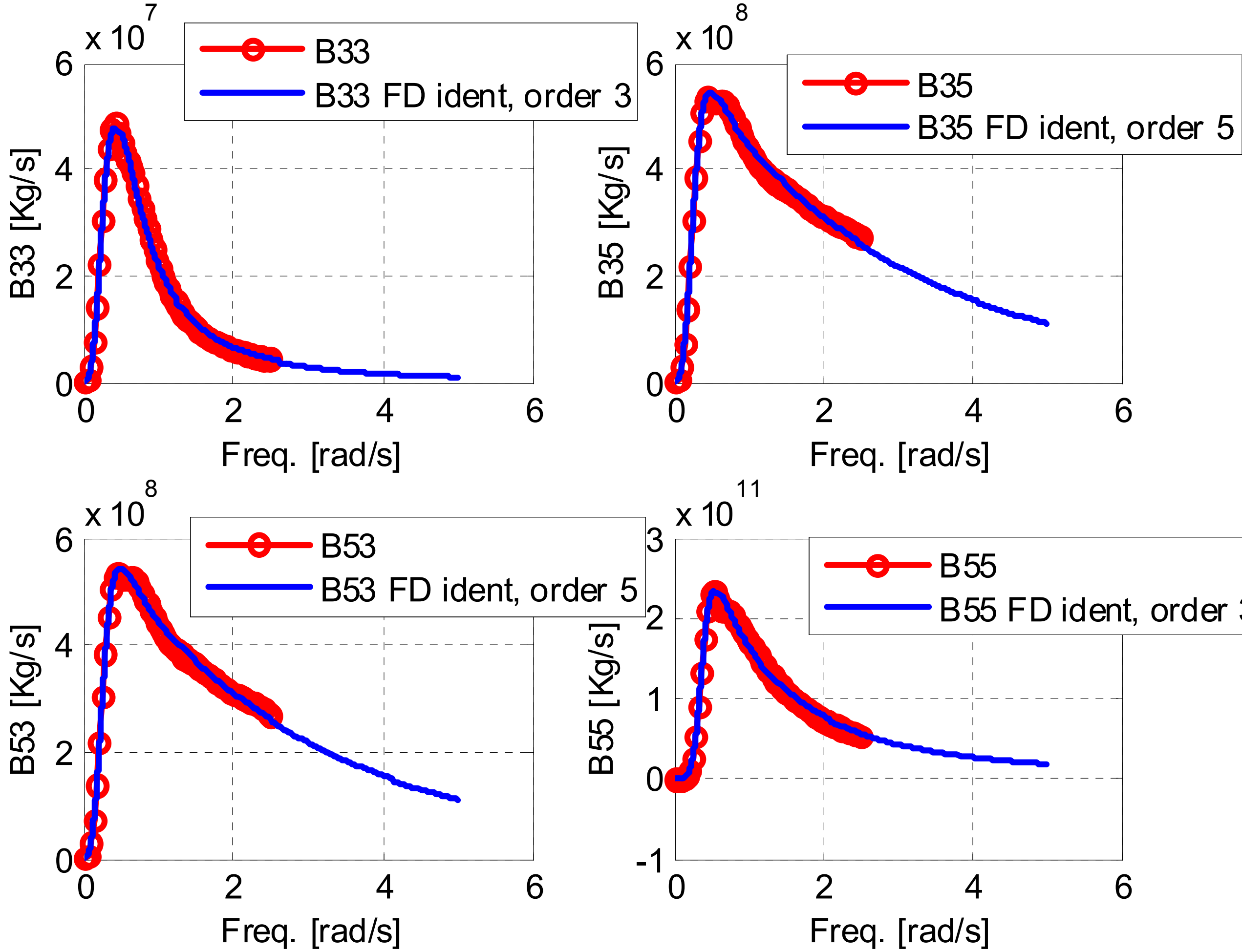
Containership example



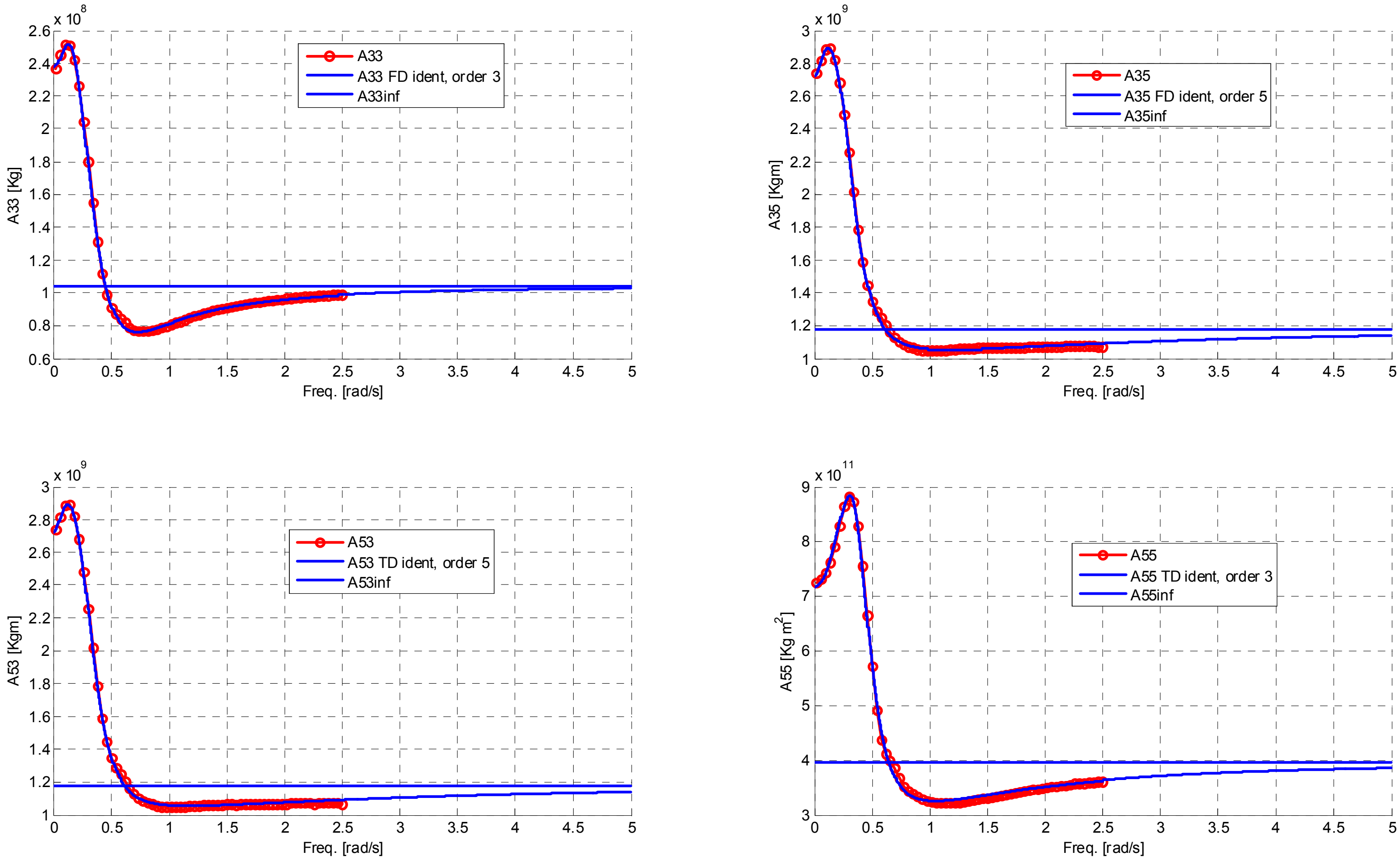
Frequency-domain identification of K35(jw) order 5 and K55(jw) order 3
Identification method: iterative quasi-linear regression (invfreqs.m)



Reconstructing B(ω)



Reconstructing A(ω)

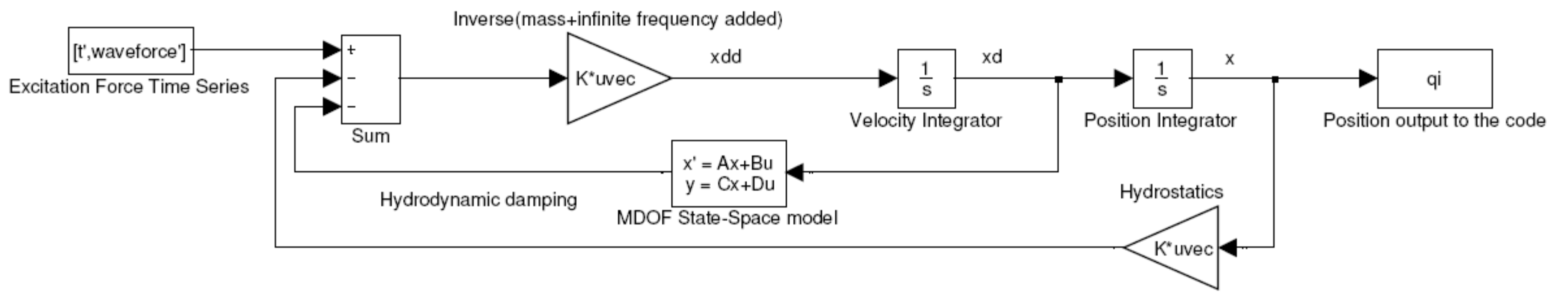


Comments about Freq.-dom. regression

- FD-identification avoids having to compute the impulse response from the damping
- The identification method is simplest: a series of linear LS problems—easy to programme.
- The zero at s=0 and the relative degree can be enforced in the structure of the model, so the asymptotic values are always ensured.
- The resulting models may be unstable: this is fixed by reflecting the unstable poles about the imaginary axis.
- The resulting models may not be passive: this can be solved using weights in the LS problem.

Simulink Model implementation

After obtaining a state-space representation or the transfers functions, we can assemble a complete model:



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