A Discussion About Seakeeping and Manoeuvring Models For Surface Vessels

Tristan Perez, Thor I. Fossen and Asgeir Sørensen

Technical Report (MSS-TR-001)

Marine System Simulator (MSS) Group
(`http://www.cesos.ntnu.no/mss/`)

Centre for Ships and Ocean Structures (CESOS)
Norwegian University of Science and Technology NTNU
NO-7491 Trondheim, NORWAY

November 12, 2004

1 Introduction

Surface vessel operations are performed under different environmental conditions, and different assumptions are made during the study of hydromechanics[1] in each case. As a consequence of this, the study of ship dynamics has traditionally been separated into two main areas:

- *manoeuvring* or controllability in calm water and
- *seakeeping* or vessel motion in a seaway.

Manoeuvring is associated, for example, with course keeping, course changes, turning, stopping, *etc*. These operations are often performed in open or restricted calm waters (*i.e.*, in calm open seas and in sheltered waters or in harbors). Seakeeping, on the other hand, is associated with motion in a seaway while the vessel keeps its course and its speed constant. These two areas of study of ship motion are well established and there are accurate models to describe the motion of the ship in each.

Despite this separation, there are operations and conditions that lie at the interface of these two areas, *e.g.*, broaching, deck diving, station-keeping and rudder roll stabilisation. The latter two require the design of control systems, which force

[1] *ship hydromechanics* refers the mechanical loads and motion produced by the interaction between the hull of the ship and the water.

the designer to combine models from the two different areas to study ship motion. This raises the question as to which way one can combine the available tools to obtain both simple models for control system design and comprehensive models for simulation and testing of new control strategies. The purpose of this note is, thus, to introduce a discussion this topic.

2 Theories of Ship Motion

2.1 Seakeeping

Seakeeping uses linear equations of motion to describe the response of the vessel to the wave excitation loads. This way, the principle of superposition holds for the responses, and ship motion can be studied using simple filtering and stochastic process theory (St Denis and Pierson, 1953).

Most of the analysis done in seakeeping is performed in the frequency domain, and the end result sought is to compare ship performance with prescribed limits in statistical terms—see. foe example, (Lloyd, 1989). The key elements of this analysis are the, so-called *Response Amplitude Operators* (RAO). These are transfer-function-like operators that that give the frequency response of the of the vessel motion to the wave amplitude as a function of the frequency. Having, these RAO, one can then combine them with the se spectrum to obtain the power spectrum of the motion components of the ship: surge, sway, heave, roll, pitch and yaw. From the power spectrum of the motion components all the necessary statistics of ship motion are obtained and the seakeeping analysis performed.

The equations of motion are given by, (Salvesen *et al.*, 1970; Faltinsen, 1990),

$$\sum_{k=1}^{6}[M_{ik} + A_{ik}(\omega_e)]\ddot{\eta}_k + B_{ik}(\omega_e)\dot{\eta}_k + C_{ik}\eta_k = \tau_{Wk} \quad \text{for} \quad i = 1, ..., 6. \tag{1}$$

where M_{ik} are the rigid body generalised mass coefficients, $A_{ik}(\omega_e)$ are the the added mass coefficients, $B_{ik}(\omega_e)$ are the potential and equivalent linearised viscous damping coefficients and C_{ik} are the linear restoring coefficients. The wave excitation forces are represented by τ^*_{Wk}. These equations are solved in the frequency domain for sinusoidal wave excitations. Indeed, using complex notation for the harmonic wave excitations and the components of motion:

$$\begin{aligned} \tau_{Wi}(t) &= \Re\left\{\tau^*_{Wi}\, e^{j\omega_e t}\right\} = \Re\left\{|\tau^*_{Wi}|\, e^{j\arg\tau^*_{Wi}}\, e^{j\omega_e t}\right\}, \\ \tilde{\eta}_i(t) &= \Re\left\{\eta^*_i\, e^{j\omega_e t}\right\} = \Re\left\{|\eta^*_i|\, e^{j\arg\eta^*_i}\, e^{j\omega_e t}\right\}, \end{aligned} \tag{2}$$

the equations of motion for sinusoidal excitation reduce to solve

$$\sum_{k=1}^{6} -\omega_e^2(M_{ik} + A_{ik}(\omega_e))\eta_k^* + j\omega_e B_{ik}(\omega_e)\eta_k^* + C_{ik}\eta_k^* = \tau_{Wk}^* \quad \text{for} \quad i = 1, ..., 6. \tag{3}$$

Equations (3) are evaluated only for a discrete set of frequencies (typically 20 to 40), and the results give the amplitude and phase of each component of motion at each frequency. Since for each frequency equations (3) are linear, the results give the amplitude and phase of each component of motion per unit of wave amplitude:

$$H_k(\omega_e, \chi) = \left| \frac{\eta_k^*(\omega_e)}{\bar{\zeta}} \right| e^{j \arg \eta_k^*(\omega_e)} \quad (\text{for} \quad k = 1, ..., 6) \tag{4}$$

These results represent the frequency response of the ship and are called the ship *Response Amplitude Operators* (RAO). The RAO can be expressed either in the wave frequency domain or in the encounter frequency domain:

$$H_k(\omega_e, \chi) \equiv H_k(\omega, U, \chi).$$

From the control engineering perspective, time series of ship motion can be generated from the motion spectrum to model disturbances. This can alternatively be done via shaping filters fitted to the motion spectrum and driven by white noise or by using the fourier representation of the stochastic process, *i.e.*, a sum of sinusoids with constant amplitude (obtained from the spectrum) and random phases (St Denis and Pierson, 1953): Indeed, by formulating the problem in the wave encounter frequency domain ω, we have that the time series for the different motion components can be generated as indicated in the following:

for $i = 1, 2, 3$

$$\eta_i(t) = \sum_{n=1}^{N} \sum_{m=1}^{M} \bar{\eta}_{inm}(\omega_n) \cos\left[\left(\omega_n - \frac{\omega_n^2 U}{g} \cos(\chi_m) \right) t + \vartheta_{inm}(\omega) + \epsilon_n \right], \tag{5}$$

for $i = 4, 5, 6$

$$\eta_i(t) = \sum_{n=1}^{N} \sum_{m=1}^{M} \bar{\eta}_{inm}(\omega_n) k_n \sin\left[\left(\omega_n - \frac{\omega_n^2 U}{g} \cos(\chi_m) \right) t + \vartheta_{inm}(\omega) + \epsilon_n \right], \tag{6}$$

with

$$\bar{\eta}_{inm}(\omega_n^*) = \sqrt{2|H_i(\omega_n^*, U, \chi_m)|^2 \mathbf{S}_{\zeta\zeta}(\omega_n^*, \chi_m^*)\Delta\chi\Delta\omega}. \tag{7}$$

$$\vartheta_{inm}(\omega) = \arg H_i(\omega_n^*, U, \chi_m^*), \tag{8}$$

and ω_n^* chosen randomly in the interval

$$[\omega_n - \frac{\Delta\omega}{2}, \omega_n + \frac{\Delta\omega}{2}],$$

where $\mathbf{S}_{\zeta\zeta}(\omega, \chi)$ is the wave directional spectrum.

This gives rise to what we call seakeeping models for disturbance and the elements of these models are depicted in Figure 1.

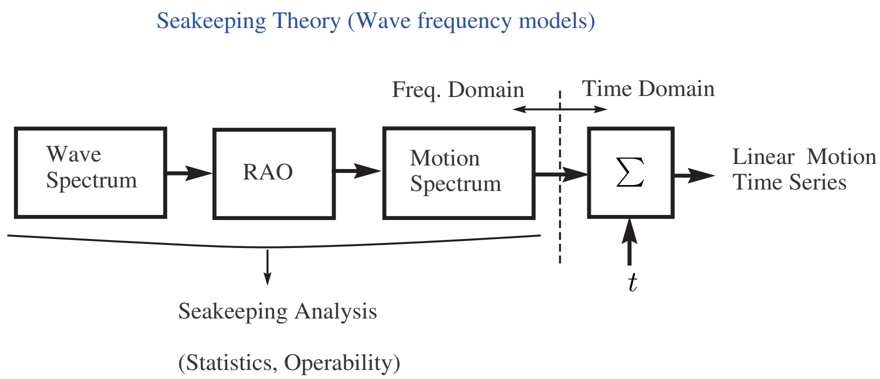

Figure 1: Ship motion models based on seakeeping theory.

To summarise, seakeeping models present the following characteristics:

- The equations of motion are described from an equilibrium frame traveling with the average forward speed of the ship and fixed at the time-average position of certain point of the ship.
- The mass and damping coefficients of the equations of motion are frequency dependant, so the equations are solved for a discrete set of frequencies, and since for a fixed frequency the equations are linear, the results give the amplitude and phase of the motion components per unit of wave amplitude as a function of the frequency–the RAO.
- Only the steady state of the motion can be computed in time domain simulations as time series.
- The models are not accurate at low frequency.

2.2 Manoeuvring

The study of manoeuvring characteristics of ships assess the effect of devises used to control the heading of the ship: rudders, thrusters, and main propulsion systems. These characteristics are studied in calm water. A model for describing motion during manoeuvring are linear and non linear equations of motion—this depends on the application, *e.g.*, models for course keeping can be described within a linear framework, but models for turning requires non linear terms.

In a vector form, the equations of motion can be expressed as (Fossen, 2002):

$$\begin{aligned} &[\mathbf{M} + \mathbf{A}(0)]\dot{\boldsymbol{\nu}} + \mathbf{C}(\boldsymbol{\nu})\boldsymbol{\nu} + D(\boldsymbol{\nu})\boldsymbol{\nu} + \mathbf{G}\boldsymbol{\eta} = \boldsymbol{\tau} \\ &\dot{\boldsymbol{\eta}} = J(\boldsymbol{\Theta})\boldsymbol{\nu}, \end{aligned} \tag{9}$$

where

- $\mathbf{M}$ is the rigid body mass matrix
- $\mathbf{A}(0)$ is the low frequency added mass matrix: $\mathbf{A}(0) = \mathbf{A}(\omega_e \to 0)$,
- $\mathbf{C}(\boldsymbol{\nu})$ is the total coriolis and centripetal accelerations matrix,
- $\mathbf{D}(\boldsymbol{\nu})$ is the total damping matrix,
- $\mathbf{G}(\boldsymbol{\eta})$ is the restoring function,
- $\mathrm{J}(\boldsymbol{\Theta})$ is the kinematic transformation
- $\boldsymbol{\tau}$ is the vector of forces and moments acting on the hull originated by the control devices and the propulsion system.

An schematic representation of these models are depicted in the bottom diagram of Figure 2. To summarise, manoeuvring models present the following characteristics:

- The equations of motion are formulated in a reference frame fixed to the ship, and not in an equilibrium frame like in seakeeping.
- These equations can be linear or non-linear depending on the application.
- The coefficients of the equations are estimated from captive scale-model tests, by measuring forces while the model is subjected to low frequency oscillations in 3DOF (surge, sway and yaw) or 4DOF (with the addition of roll).
- The linear coefficients—hydrodynamic derivatives—take the asymptotic values of the the coefficients used in the seakeeping equations of motion in the limit as $\omega \to 0$.

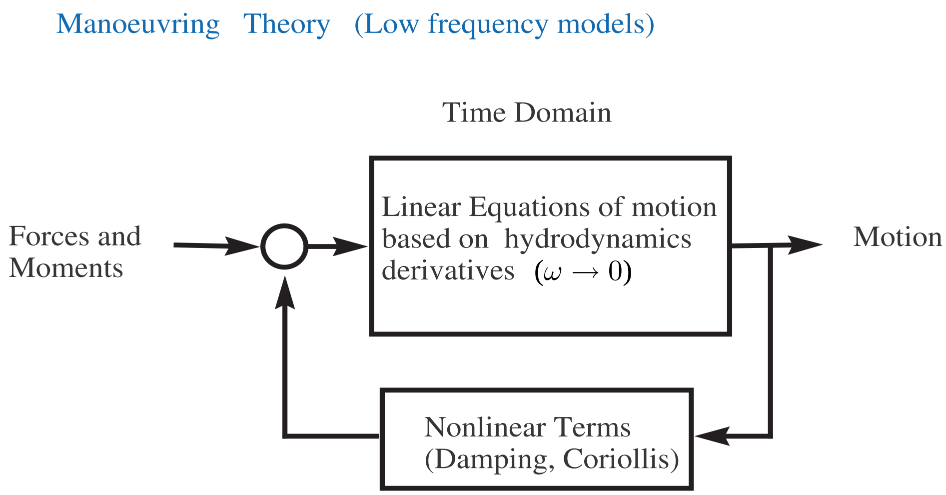

Figure 2: Ship motion models based on manoeuvring theory.

3 Models for Control that Require Manoeuvring in Seaway

The typical manner to combine the models for control system design is to use the seakeeping model as an motion output disturbance of a manoeuvring model that captures the interaction between the control action and the motion generated by this control action. This schemme is depicter in Figure 3.

A first shortcoming associated with the manoeuvring model to describe motion in waves is that the added mass and damping are constant in this model; and thus, memory effects are neglected. From a control perspective this introduces some uncertainty associated with un-modelled dynamics, which for operations in higher sea states could be significant; specially if a model based control strategy is applied. A second shortcoming is that the output disturbance modelling approach can only be used to study operations with a single ship. This is because the interaction between different dynamic systems require energy exchange, *i.e.*, it requires common forces or speeds, and this is not captured by models that use motion as a disturbance. This issue is relevant to marine operations.

A desirable extension is to use a model with wave loads excitation forces as inputs and memory effects in the equations of motion, as indicated in Figure 4. In this figure, FTF represents the wave excitation forces per unit of wave amplitude:

$$FTF_k(\omega) = \left| \frac{\tau^*_{Wk}(\omega)}{\bar{\zeta}} \right| e^{j \arg \tau^*_{Wk}(\omega)} (\text{for} \quad k = 1, ..., 6), \tag{10}$$

which are obtained from any standard seakeeping software–see, for example (Fathi, 2004; Jouernee and Adegeest, 2003).

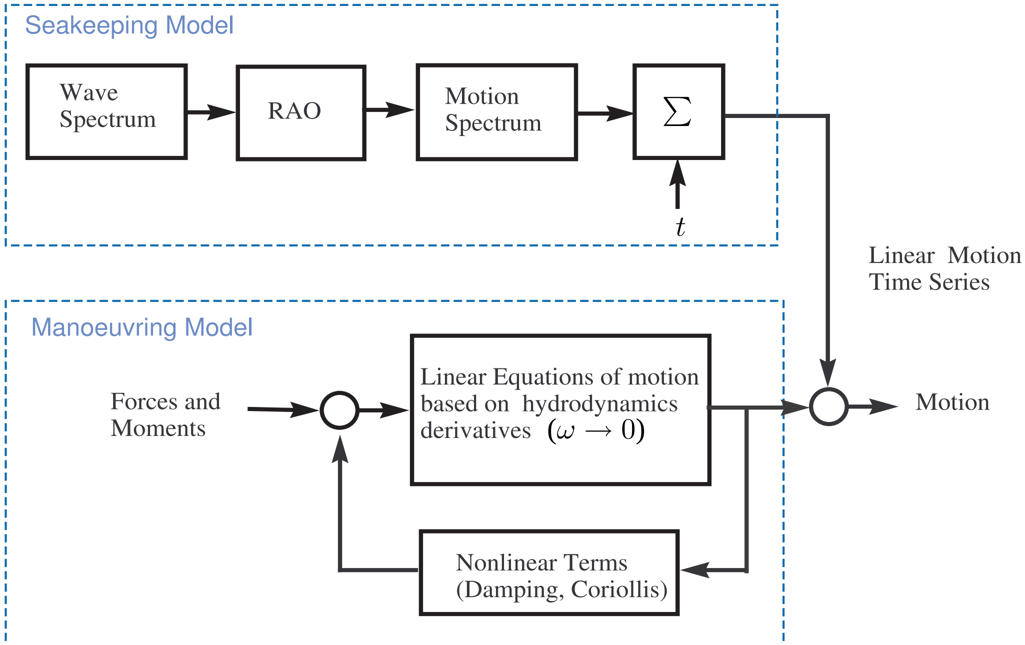

Figure 3: Typical model for ship motion control system design.

The models depicted in Figure 4 are part of the state of art in time-domain ship motion simulators, but have not yet been employed in control system design. However, recent reported results suggest that this is a feasible task (Kristansen and Egeland, 2003; Fossen and Smogeli, 2004).

Cummins (1962) showed that the frequency dependent terms in (9), $\mathbf{A}(\omega_e)$ and $\mathbf{B}(\omega_e)$ can be removed by writing the equations of motion in the following form:

$$[\mathbf{M} + \mathbf{A}(\infty)]\dot{\boldsymbol{\nu}} + \int_{-\infty}^{t} \mathbf{K}(t-\tau)\boldsymbol{\nu}(\tau)\, d\tau + \mathbf{G}\eta = \boldsymbol{\tau}_W + \boldsymbol{\tau} \tag{11}$$

From Ogilvie (1964) it follows that:

$$\mathbf{A}(\infty) = \mathbf{A}(\infty)^T = \lim_{\omega_e \to 0} \mathbf{A}(\omega_e), \qquad \dot{\mathbf{A}}(\infty) = \mathbf{0} \tag{12}$$

$$\mathbf{K}(\tau) = \frac{2}{\pi}\int_0^{\infty} [\mathbf{B}(\omega_e) - \mathbf{B}(\infty)] \cos(\omega_e \tau)\, d\omega_e \tag{13}$$

$$\mathbf{K}(\tau) = -\frac{2}{\pi}\int_0^{\infty} \omega_e[\mathbf{A}(\omega_e) - \mathbf{A}(\infty)] \sin(\omega_e \tau)\, d\omega_e \tag{14}$$

where $\mathbf{A}(\infty)$ is a constant generalized added mass matrix evaluated at the infinity frequency, $\mathbf{K}(\tau) \in \mathbb{R}^{6\times 6}$ is a time-varying matrix of *retardation functions* which can be computed off-line using the $A(\omega_e)$ or $B(\omega_e)$ data sets and (13)–(14), since $\mathbf{K}(\tau)$ is causal.

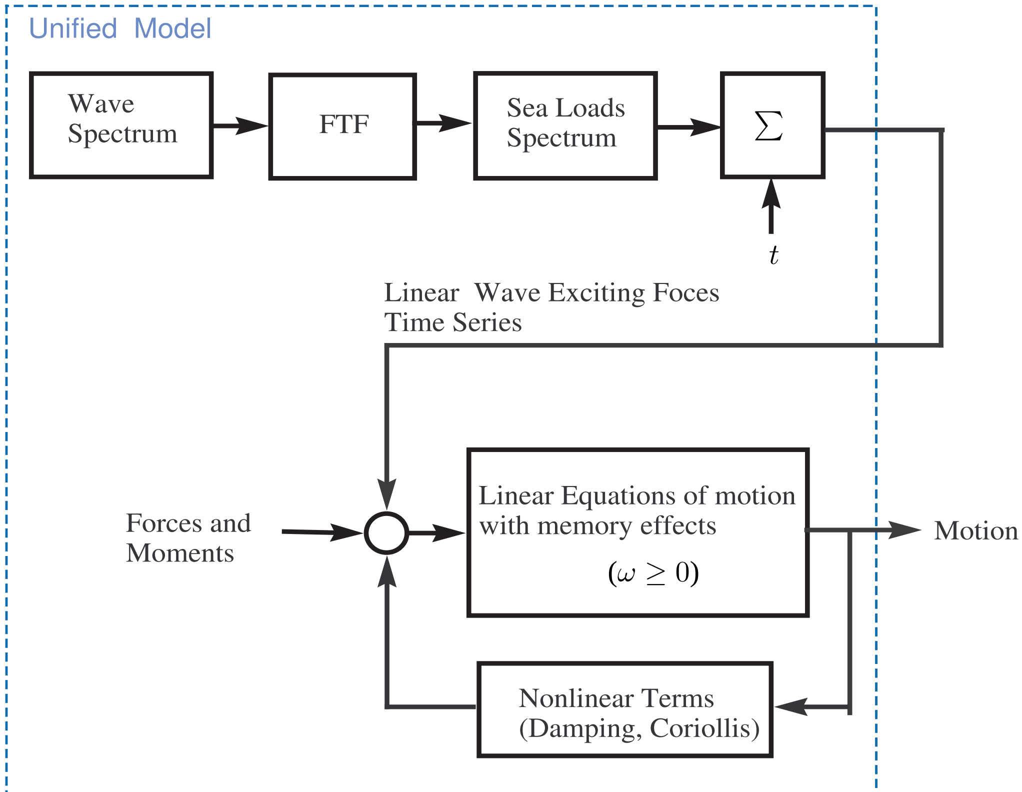

Figure 4: Unified model for manoeuvring in seaway.

Kristansen and Egeland (2003) developed a state-space formulation for the potential damping term in (11). Consider:

$$\boldsymbol{\mu}(t) = \int_{-\infty}^{t} \mathbf{K}(t-\tau)\boldsymbol{\nu}(\tau)d\tau \overset{\text{causal}}{=} \int_{0}^{t} \mathbf{K}(t-\tau)\boldsymbol{\nu}(\tau)d\tau \tag{15}$$

where $\mathbf{K}(t-\tau)$ is the *retardation function.* For causal systems:

$$\mathbf{K}(t-\tau) = \mathbf{0} \text{ for } t < 0 \tag{16}$$

If $\boldsymbol{\nu}(\tau)$ as a unit impulse, then $\mu(t)$ given by (15) will be an *impulse response* function. Consequently, $\mu(t)$ can be represented by a linear state-space model:

$$\dot{\boldsymbol{\chi}} = \mathbf{A}_r\boldsymbol{\chi} + \mathbf{B}_r\boldsymbol{\nu}, \qquad \boldsymbol{\chi}(0) = \mathbf{0} \tag{17}$$

$$\boldsymbol{\mu} = \mathbf{C}_r\boldsymbol{\chi} + \mathbf{D}_r\boldsymbol{\nu} \tag{18}$$

where $(\mathbf{A}_r, \mathbf{B}_r, \mathbf{C}_r, \mathbf{D}_r)$ are constant matrices of appropriate dimensions. Using this, the following time-domain model with memory effect is obtained for the case of zero forward speed (Fossen and Smogeli, 2004):

$$\dot{\boldsymbol{\eta}} = J(\Theta)\boldsymbol{\nu} \tag{19}$$

$$[M + A_\infty]\dot{\boldsymbol{\nu}} + \boldsymbol{\mu} + G\boldsymbol{\eta} = \boldsymbol{\tau}_W + \boldsymbol{\tau} \tag{20}$$

$$\dot{\chi} = \mathbf{A}_r\chi + \mathbf{B}_r\boldsymbol{\nu}, \qquad \chi(0) = \mathbf{0} \tag{21}$$

$$\boldsymbol{\mu} = \mathbf{C}_r\chi + \mathbf{D}_r\boldsymbol{\nu} \tag{22}$$

This result can be extended to the case of forward speed, but this technicalities are outside the scope of this report.

The above model presents the following properties:

- It is in a state-space form and therefore amenable to use in control system design and in common simulation tools like Matlab and Simulink.
- Includes memory effects, and this allows one to use the wave excitation forces as inputs together with the control actions. This is in agreement with the physics of the motion problem.
- It allows the simulation of the interaction of multiple vessels, because the interacting forces can be included.
- Due to the memory effects, the model is valid for different sea states, and not for calm water as the manoeuvring model.

References

Cummins, W.E. (1962). The impulse response function and ship motion.. *?*

Faltinsen, O.M. (1990). *Sea Loads on Ships and Offshore Structures*. Cambridge University Press.

Fathi, D. (2004). *ShipX Vessel Responses (VERES)*. Marintek AS Trondheim. http://www.marintek.sintef.no/.

Fossen, T.I. (2002). *Marine Control Systems, Guidance, Navigation and Control of Ships, Rigs and Underwater Vehicles*. Marine Cybernetics.

Fossen, T.I and Ø.N. Smogeli (2004). Nonlinear time-doamin strip theory formulation for low speed manoeuvring and station-keeping.. *Modelling Identification and Control–MIC*.

Jouernee, J.M.J. and L.J.M. Adegeest (2003). *Theoretical Manual of Strip Tehory Program SEAWAY for Windows*. TU Delft, Delft Unoversity of Technology. www.ocp.tudelft.nl/mt/journee.

Kristansen, E. and O. Egeland (2003). Frequency dependent added mass in models for controller design for wave motion ship damping.. In: *6th IFAC Conference on Manouvering and Control of Marine Craft MCMC'03, Girona, Spain.*

Lloyd, A.R.J.M. (1989). *Seakeeping: Ship Behaviour in Rough Weather*. Ellis Horwood Series in Marine Technology. Ellis Horwood.

Ogilvie, T. (1964). Reecent progress towards the undestanding and prediction of ship motions. In: *6th Symp. on Naval Hydrodynamics.*

Salvesen, N., E.O. Tuck and O.M. Faltinsen (1970). Ship motions and sea loads. *Trans. The Society of Naval Architects and Marine Engineers–SNAME* **10**, 345–356.

St Denis, M. and W.J. Pierson (1953). On the motion of ships in confused seas. *SNAME Transactions* **61**, 280–332.